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# THE GROWTH AND DECAY OF PHOTO-THERMIONIC CURRENTS FROM OXIDE COATED FILAMENTS

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T. W. Case[1] and E. Merritt[2] have recently described experiments in which the oxide coated filament of an audion is subjected to illumination, with the result of increasing the current between filament and plate. In the experiments of Prof. Merritt this added space current was shown to be principally due to short wave-length radiation, and was explained as a true photo-electric effect, that is, the direct emission of electrons due to illumination. He found the added current to be dependent upon the filament temperature. This might be taken to indicate that the emission of photo-electrons is a function of the temperature of the surface, which disagrees with conclusions drawn from experiments upon other photo-electrically active materials.

In some recent experiments we have found this effect to have characteristics sufficiently different from those of the true photo-electric effect to raise doubts as to the identity of the two phenomena. If they are not identical then the new effect is not necessarily evidence for a dependence of the photo-electric effect on temperature.

The experiments here reported have to do with the rate of growth and decay of the light induced additions to the space currents. These were recorded on a string galvanometer through a distortionless amplifier system. In every case voltages were high enough to carry across all electrons liberated. A carbon arc lamp with glass condensing lens was used as a light source, and between the condenser and the oxide coated filament was interposed either a deep red or a deep blue glass (transmitting

no red), selected so that the light transmitted by each gave added space currents of the same order of magnitude.

With the deep red glass interposed the added space currents obtained upon brief illumination with various heating currents through the filament are shown in figure 1. The effect increases regularly with temperature, but takes a considerable time to rise and fall at the beginning and end of the period of illumination. The times of growth and decay depend upon the magnitude of the current, and the curves have the general appearance of heating and cooling curves. We believe this red light effect to be chiefly due to the heating of the filament by the incident radiation. Support of this belief is given by figure 2, in which the same increase of space current is caused, first (a) by red light, and second (b) by a sudden brief increase of filament heating current. It will be seen that the two curves are similar. We have also found that the added space current varies with filament current in the way that it would for equal added filament-power increments; and that the change of resistance of the filament is approximately the same whether the space current increment is due to red light or filament current.

With the blue glass, the energy transmission, as measured by a thermocouple, was only a few per cent that of the red glass, although for a chosen filament temperature near the middle of the available range both transmitted radiations gave the same added space current. The heating effect of the blue radiation is therefore negligible, and the added space current may be considered as a true light effect. String galvanometer records of the added space current due to brief blue illumination, for various filament currents (temperatures) are reproduced in figure 3. It will be seen at once that the added space currents exhibit a behavior essentially different from those due to red light or heating. Whereas with red light the current continually increases with temperature, with blue light the added current at first increases and then starts to decrease. At low temperatures the growth and decay are exceedingly slow. As the temperature is raised the rates of growth and decay increase until at the highest temperature studied the response appears practically instantaneous.

It is the existence of this slow response and its variation with temperature that we believe differentiate this effect from the true photo-electric effect, which, as far as previously known (and in agreement with tests made by us on a potassium cell) is instantaneous at all temperatures.

The true nature of the increased electronic emission under blue illumination is not decided by our experiments, but several possibilities suggest themselves. It may be a light induced increase in the number of electrons available for thermionic emission; or it may be a light induced change

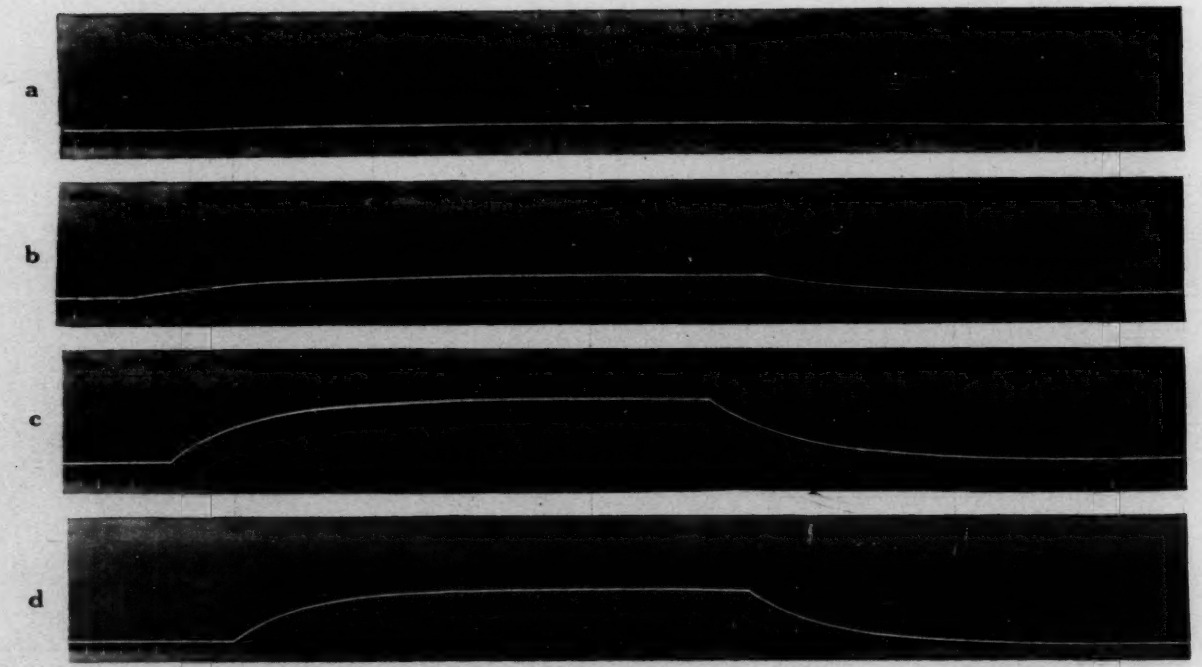


FIGURE 1

Added space currents due to illumination of audion filament by red light. (a) Filament heating current = .65 amp.; (b) filament heating current = .75 amp.; (c) filament heating current = .85 amp.; (d) filament heating current = .95 amp.; (sensibility of galvanometer reduced to half.) Larger time divisions = 0.1 second.

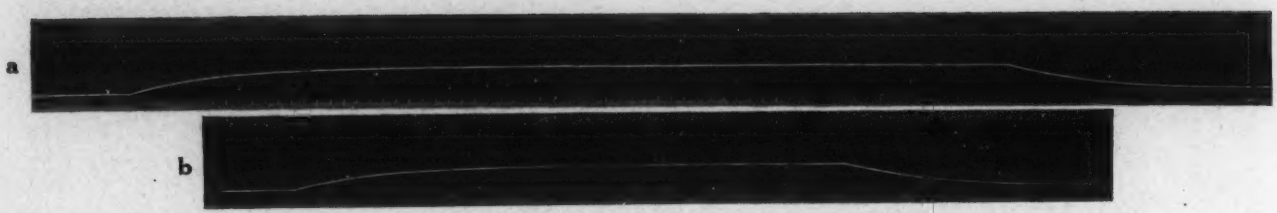


FIGURE 2

Equal added space currents caused by (a) illumination by red light; (b) increase of heating current.

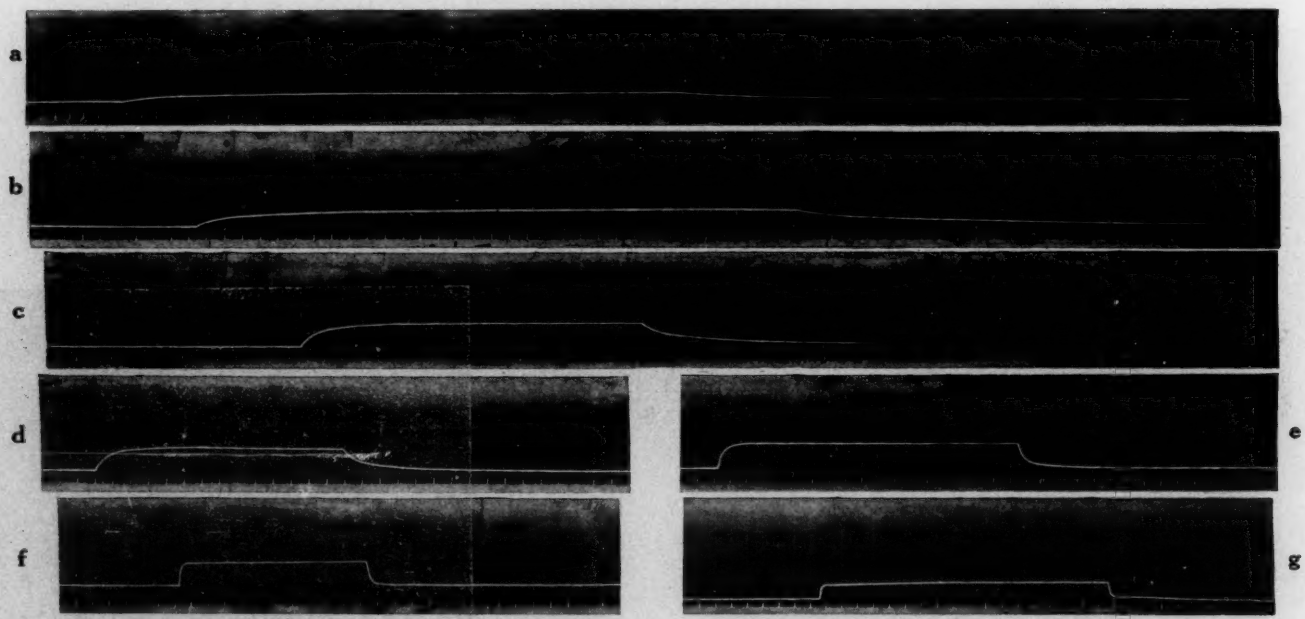


FIGURE 3

Added space currents due to illumination of audion filament by blue light. Filament heating currents = (a) .60 amp.; (b) .64 amp.; (c) .70 amp.; (d) .75 amp.; (e) .80 amp.; (f) .87 amp.; (g) .94 amp.
Filament temperature range 500° C. to 800° C. Larger time divisions = 0.1 second.

in the work function of the surface, so that more of the electrons present may escape;[3] or it may be a light induced change in the chemical nature of the surface. In any case there must be ascribed to the light produced change a temperature coefficient of speed of response. Some clue to the nature of the phenomenon may perhaps be obtained from a study of the growth and decay curves. They are not simple exponential curves such as would represent the charging and discharging of a condenser. If the reciprocal of the square root of the decaying current increment is plotted against time, the plots are very nearly straight lines for all temperatures (law of phosphorescence decay). Attention may be called to the fact that the growth and decay of current represented in the curves in figure 3 exhibiting slow response at low temperatures and quick response at high, are quite similar to the growth and decay of current in selenium under illumination, which likewise show a great variation of speed with temperature. This analogy suggests that the cause of the light effect in the oxide coated filament may be closely related to that which gives selenium its photo-sensitive properties. It should also be noted that the change of speed of response with temperature would be expected if one assumed the effect to be due to a chemical reaction.

In view of the data presented it is believed that this photo-effect in thermionic filaments cannot safely be considered as evidence for a variation of the true photo-electric effect with temperature. We suggest the use of the term "photo-thermionic emission."

[1] T. W. Case, *Physic. Rev.*, 17, 3, p. 398.

[2] E. Merritt, *Ibid.*, 17, 4, p. 525.

[3] Koppius (*Ibid.*, Mar. 1921, p. 395) found with oxide coated filaments a decrease in the work function with increasing temperatures.

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## *GROUP OF ISOMORPHISMS OF A TRANSITIVE SUBSTITUTION GROUP*

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Let $G$ represent any transitive substitution group of degree $n$. It is convenient to divide the possible automorphisms of $G$ into three categories. First, those which can be obtained by transforming all the substitutions of $G$ successively by each of its own substitutions. In this way the group of inner isomorphisms of $G$ can be found and this group is known to be an invariant subgroup of the group of isomorphisms $I$ of $G$. The second category consists of the automorphisms obtained by transforming $G$ by substitutions on its own letters, which transform $G$ into

itself but are not found in $G$. It will be proved that these automorphisms together with the inner ones constitute all the possible automorphisms of $G$ in which the subgroups composed separately of all the substitutions of $G$ omitting a particular letter correspond to such subgroups.

The third category of automorphisms of $G$ is composed of all those in which the subgroups composed of all the substitutions of $G$ which omit a given letter correspond to subgroups of degree $n$. Such automorphisms are not always possible. In fact it may happen that $I$ is simply isomorphic with the group of inner isomorphisms of $G$. In this case only the first category of automorphisms actually exists. A necessary and sufficient condition that the first category of automorphisms reduces to the identity is that $G$ is abelian.

To prove the theorem noted at the end of the first paragraph we may consider any possible automorphism of $G$ in which the subgroup $G_1$ composed of all the substitutions of $G$ which omit a given letter $a$ corresponds to itself while some of the substitutions of this subgroup do not necessarily correspond to themselves. In this automorphism every substitution of $G$ which involves the letter $a$ must correspond to such a substitution, and we shall suppose that the substitutions are represented in such a way that this letter appears first.

There are $g/n$, $g$ being the order of $G$, substitutions of $G$ in which the letter $a$ is followed by any other letter $b$. If in one of the corresponding substitutions under the said automorphisms $a$ is followed by $d$ then this must be the case in each of the $g/n$ substitutions which correspond to the substitutions beginning with the letters $ab$. If this were not the case as regards some two such corresponding substitutions then the inverses of the former of the two given corresponding substitutions into the latter would give two corresponding substitutions of which the former would omit the letter $b$ while the latter would involve the letter $d$. This is impossible since the subgroup composed of all the substitutions which omit $b$ must correspond to the subgroup composed of all the substitutions which omit $d$ in the automorphism under consideration as a result of the fact that a substitution beginning with $ab$ corresponds to one beginning with $ad$.

From the preceding paragraph it results that the second letters in all of the corresponding substitutions involving the letter $a$ in the said automorphism determine a single substitution $t$ on the letters of $G$. This substitution does not involve the letter $a$. It transforms $G$ into a simply isomorphic group $G'$ such that in the corresponding substitutions involving $a$ the second letters are exactly the same as they are in the second group of the automorphism under consideration. If $G$ were not identical with this second group, as a substitution group, it would be possible to establish a simple isomorphism between $G$ and another substitution group $G'$ such that every substitution involving $a$ would correspond to such a

substitution and that the second letter in all these corresponding substitutions would be the same in every case.

In this simple isomorphism every two corresponding substitutions would be identical. For, if $S_1$, $S_1'$ are any two corresponding substitutions such that in $S_1$ the letter $e$ is followed by the letter $f$, and if $S_2$, $S_2'$ are two other corresponding substitutions in which the letter $a$ is followed by $e$ then $S_1$ is obtained by multiplying the inverse of $S_2$ into one of the $g/n$ substitutions in which $a$ is followed by $f$. Hence it results that in $S_1'$ the letter $e$ is also followed by the $f$. That is, $S_1$ and $S_1'$ are identical. In this proof it was assumed that $e$ is not $a$ and that $f$ is not $a$. If $e$ were $a$ then $a$ would be replaced by $f$ in $S_1'$ by hypothesis. If $f$ were $a$ then in the inverse of $S_1$ the letter $a$ would be replaced by $e$ and hence in the inverse of $S_1'$ the letter $a$ would be replaced by $e$. Therefore the theorem again requires no further proof. Hence it has been established that $G$ and $G'$ are identical, and that the automorphism under consideration can be effected by transformed $G$ by the substitution $t$ which therefore transforms $G$ into itself.

If the subgroup $G_1$ had corresponded in the given automorphism to a conjugate of $G_1$ but different from $G_1$ there would be a substitution in $G$ which would transform $G_1$ into this conjugate. The inverse of this substitution into $t$ would transform $G$ into itself and effect the automorphism in question. It has therefore been proved that *every automorphism of $G$ in which the subgroups composed of all the substitutions of $G$ which omit a given letter correspond to such subgroups is effected by substitutions of the largest group on the letters of $G$ which contains $G$ invariantly.* As a particular case of this theorem we have the well known result that when $G$ is regular it is transformed into all its possible automorphisms under its holomorph.

From the theorem proved in the preceding paragraph it is easy to deduce a method for finding the order of the group of isomorphisms of the transitive substitution group $G$. If $G_1$ is of degree $n - \alpha$ the largest group on the letters of $G$ which contains $G$ invariantly involves exactly $\alpha$ substitutions which are commutative with every substitution of $G$ and hence this largest group transforms the substitutions of $G$ according to a group whose order is the order of this largest group divided by $\alpha$. This group of transformation is simply isomorphic with the quotient group of this largest group with respect to its invariant subgroup composed of the $\alpha$ substitutions which are commutative with every substitution of $G$. The order of the $I$ of $G$ is therefore equal to the order of this quotient group multiplied by the number of the different sets of conjugate subgroups of $G$ which are such that each set involves a subgroup which corresponds to $G_1$ in some possible automorphism of $G$.

A necessary and sufficient condition that $G$ contains more than one

such set of conjugate subgroups is that it contains subgroups of degree $n$ and of index $n$ with respect to which it can be represented as a transitive substitution group which is identical with $G$. In particular, *if a given abstract group is simply isomorphic with only one group in a complete list of transitive groups of degree n then this transitive group has outer isomorphisms if it contains a subgroup of degree n and of index n which is non-invariant and does not involve any invariant subgroup of the entire group besides the identity.* From this special but useful theorem it follows directly that the symmetric group of degree 6 has outer isomorphisms. Various writers established this fact by somewhat laborious special methods.[1] It also follows directly from this theorem that the largest imprimitive groups on six letters which involve 2 and 3 systems of imprimitivity, respectively, have isomorphisms which cannot be obtained by transforming these groups by substitutions on their own letters.

While the group of inner isomorphisms is an invariant subgroup of $I$ whenever it does not coincide with $I$ it should not be inferred that the subgroup of $I$ which corresponds to all the automorphisms of $G$ which can be obtained by transforming $G$ by substitutions on its own letters is always an invariant subgroup of $I$. In fact, this is not the case when $G$ is the generalized dihedral group of order 16 involving the abelian group of type (2, 1) represented as a transitive group of degree 8. When $G$ admits automorphisms in which $G_1$ corresponds to a subgroup of degree $n$ the conjugates of $G_1$ are transformed under $I$ according to an imprimitive substitution group. One of the systems of imprimitivity of this group is composed of the conjugates of $G_1$ under $G$.

[1] Cf. O. Hölder, *Mathematische Annalen*, **46**, 1895 (345); W. Burnside, *Theory of Groups of Finite Order*, 1911, p. 209.

## *EINSTEIN STATIC FIELDS ADMITTING A GROUP $G_2$ OF CONTINUOUS TRANSFORMATIONS INTO THEMSELVES*

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1. For static phenomena in the Einstein theory the linear element of the space-time continuum can be taken in the form $V^2dx_0^2-ds^2$, where

$$ds^2 = \Sigma\, a_{ik}\, dx_i\, dx_k \ (i, k = 1, 2, 3) \qquad (1)$$

is the linear element of the physical space $S$, and the functions $V$ and $a_{ik}$ are independent of $x_0$, the coördinate of time. In this paper we determine the functions $V$ and $a_{ik}$ satisfying the Einstein equations $B_{ik} = O$ and such that the space $S$ admits a continuous group $G_2$ of transformations into itself. Bianchi[1] has shown that any 3-space admitting such a group

is of one of two types:

(I) $ds^2 = dx_1^2 + \alpha dx_2^2 + 2\beta dx_2 dx_3 + \gamma dx_3^2$,

$\alpha$, $\beta$, $\gamma$ being functions of $x_1$ alone, the operators of the group being

$$X_1 = \partial/\partial x_2,\ X_2 = \partial/\partial x_3;$$

(II) $ds^2 = dx_1^2 + \alpha dx_2^2 + 2(\beta - \alpha x_2)\, dx_2 dx_3 + (\alpha x_2^2 - 2\beta x_2 + \gamma) dx_3^2$,

$\alpha$, $\beta$, $\gamma$ being functions of $x_1$ alone, the operators of the group being

$$X_1 = \partial/\partial x_3,\ X_2 = e^{x_3}\partial/\partial x_2.$$

2. By definition

$$B_{ik} = \sum_0^3 h \left\{ ih, hk \right\} = \sum_0^3 h \left[ \frac{\partial}{\partial x_k} \left\{ \begin{matrix} ih \\ h \end{matrix} \right\} - \frac{\partial}{\partial x_h} \left\{ \begin{matrix} ik \\ h \end{matrix} \right\} + \sum_0^3 e \left( \left\{ \begin{matrix} ih \\ e \end{matrix} \right\} \left\{ \begin{matrix} ke \\ h \end{matrix} \right\} - \left\{ \begin{matrix} lh \\ h \end{matrix} \right\} \left\{ \begin{matrix} ik \\ e \end{matrix} \right\} \right) \right], \tag{2}$$

where the Christoffel quantities are calculated with respect to the linear element of the space-time continuum. When this linear element is taken in the form $V^2 dx_0^2 - ds^2$ as in § 1, the equations $B_{ik} = 0$ reduce to the six equations[2]

$$\frac{\partial^2 V}{\partial x_i \partial x_k} = \sum_1^3 e \left\{ \begin{matrix} ik \\ e \end{matrix} \right\} \frac{\partial V}{\partial x_e} - \bar{B}_{ik} V \ (i, k = 1, 2, 3), \tag{3}$$

and

$$\sum_{i,k} a^{ik} \bar{B}_{ik} = 0, \tag{4}$$

where $a^{ik}$ is the cofactor of $a_{ik}$ in the determinant of the quantities $a_{ik}$ divided by this determinant; the Christoffel symbols are calculated with respect to (1), and

$$\bar{B}_{ik} = \sum_1^3 h \left\{ ih, hk \right\} = \sum_1^3 h \sum_1^3 l\, a^{hl} \left( il, hk \right). \tag{5}$$

By assuming that $S$ is referred to a triply-orthogonal system, so that the linear element may be written $ds^2 = \sum_1^3 i\, a_i dx_i^2$ (which is possible in any 3-space[3]), and making use of the relations

$$(il, hk) = -(li, hk) = -(il, kh) = (li, kh),$$

we find that when $\bar{B}_{ik} = 0$, then all the functions $(il, hk)$ are equal to zero. This being the condition that the space $S$ is euclidean, we have the theorem:

*A necessary and sufficient condition that a 3-space be euclidean is that the functions* $\bar{B}_{ik} = 0$.

3. If we put

$$\delta = \alpha\gamma - \beta^2,\ A = \beta'\gamma - \beta\gamma',\ B = \gamma'\alpha - \gamma\alpha',\ C = \alpha'\beta - \alpha\beta',$$

(where primes indicate differentiation with respect to $x_1$) we find that all of the Christoffel quantities for the form (I) vanish except the following:

$$\left\{\begin{matrix}22\\1\end{matrix}\right\} = -\frac{\alpha'}{2},\ \left\{\begin{matrix}23\\1\end{matrix}\right\} = -\frac{\beta'}{2},\ \left\{\begin{matrix}33\\1\end{matrix}\right\} = -\frac{\gamma'}{2},\ \left\{\begin{matrix}13\\2\end{matrix}\right\} = \frac{A}{2\delta},$$

$$\left\{\begin{matrix}12\\2\end{matrix}\right\} = \frac{1}{4\delta}\left(\delta' - B\right),\ \left\{\begin{matrix}12\\3\end{matrix}\right\} = -\frac{C}{2\delta},\ \left\{\begin{matrix}13\\3\end{matrix}\right\} = \frac{1}{4\delta}\left(\delta' + B\right).$$

Also we find

$$\overline{B}_{11} = \frac{\delta''}{2\delta} - \frac{1}{4}\frac{\delta'^2}{\delta^2} - \frac{1}{2\delta}\left(\alpha'\gamma' - \beta'^2\right),\ \overline{B}_{12} = \overline{B}_{13} = 0,$$

$$\overline{B}_{22} = \frac{1}{2}\alpha'' - \frac{\alpha'}{4}\frac{\delta'}{\delta} + \frac{\alpha}{2\delta}\left(\alpha'\gamma' - \beta'^2\right),$$

$$\overline{B}_{23} = \frac{1}{2}\beta'' - \frac{\beta'}{4}\frac{\delta'}{\delta} + \frac{\beta}{2\delta}\left(\alpha'\gamma' - \beta'^2\right),$$

$$\overline{B}_{33} = \frac{1}{2}\gamma'' - \frac{\gamma'}{4}\frac{\delta'}{\delta} + \frac{\gamma}{2\delta}\left(\alpha'\gamma' - \beta'^2\right).$$

Equation (4) reduces to

$$2\alpha\gamma'' + 2\alpha''\gamma + 3\left(\alpha'\gamma' - \beta'^2\right) - 4\beta\beta'' - \frac{\delta'^2}{\delta} = 0. \qquad (6)$$

The problem reduces to the integration of (3), when $\alpha$, $\beta$, $\gamma$ are subject to the condition (6). We separate the discussion into the four cases, when $V$ is independent of $x_2$ and $x_3$; of either $x_2$ or $x_3$; of $x_1$; involves all three variables. Expressing the conditions of integrability of (3) we are led to linear partial differential equations of the first order and these are ultimately solved. The first and second cases are the only ones giving solutions; in the course of the investigation it is shown that in both cases a change of variables can be made so that $\beta = 0$. The solutions of the first type are:

$$\alpha = x_1^{\frac{2(k+1)}{k^2+k+1}},\ \gamma = x_1^{\frac{2k(k+1)}{k^2+k+1}},\ V = x_1^{-\frac{k}{k^2+k+1}}, \qquad (7)$$

where $k$ is any constant. Solutions of this kind have been found by Kasner.[4] For the second case the linear element of $S$ may be written

$$ds^2 = \frac{d\alpha^2}{b\alpha + a\alpha^{1/2}} + \alpha dx_2^2 + \left(b + a\alpha^{-\frac{1}{2}}\right)dx_3^2, \qquad (8)$$

and

$$V = \alpha^{1/2}\varphi(x_2), \qquad \frac{d^2\varphi}{dx_2^2} + b\varphi = 0, \qquad (9)$$

where $a$ and $b$ are constants. This is the solution found by Levi-Civita[5]

and called the *quadrantal* solution.

4. For the linear element (II) the expressions for the Christoffel symbols which are different from zero are:

$$\left\{\begin{matrix}22\\1\end{matrix}\right\} = -\frac{\alpha'}{2}, \left\{\begin{matrix}23\\1\end{matrix}\right\} = \frac{1}{2}\left(\alpha' x_2 - \beta'\right),$$

$$\left\{\begin{matrix}33\\1\end{matrix}\right\} = -\frac{1}{2}\left(\alpha' x_2^2 - 2\beta' x_2 + \gamma'\right),$$

$$\left\{\begin{matrix}12\\2\end{matrix}\right\} = \frac{1}{4\delta}\left(-2C x_2 - B + \delta'\right), \left\{\begin{matrix}13\\2\end{matrix}\right\} = \frac{1}{2\delta}\left(C x_2^2 + B x_2 + A\right),$$

$$\left\{\begin{matrix}22\\2\end{matrix}\right\} = -\frac{\alpha}{\delta}\left(\alpha x_2 - \beta\right), \left\{\begin{matrix}23\\2\end{matrix}\right\} = \frac{(\alpha x_2 - \beta)^2}{\delta},$$

$$\left\{\begin{matrix}33\\2\end{matrix}\right\} = \frac{\beta - \alpha x_2}{\delta}\left(\alpha x_2^2 - 2\beta x_2 + \gamma\right),$$

$$\left\{\begin{matrix}12\\3\end{matrix}\right\} = -\frac{C}{2\delta}, \left\{\begin{matrix}13\\3\end{matrix}\right\} = \frac{1}{4\delta}\left(2C x_2 + B + \delta'\right), \left\{\begin{matrix}22\\3\end{matrix}\right\} = -\frac{\alpha^2}{\delta},$$

$$\left\{\begin{matrix}23\\3\end{matrix}\right\} = \frac{\alpha(\alpha x_2 - \beta)}{\delta}, \left\{\begin{matrix}33\\3\end{matrix}\right\} = -\frac{(\alpha x_2 - \beta)^2}{\delta}.$$

The expressions for the functions $\overline{B}_{ik}$ are

$$\overline{B}_{11} = \frac{\delta''}{2\delta} - \frac{1}{4}\frac{\delta'^2}{\delta^2} - \frac{1}{2\delta}\left(\alpha'\gamma' - \beta'^2\right), \quad \overline{B}_{12} = \frac{C}{\delta},$$

$$\overline{B}_{13} = -\frac{1}{2\delta}\left(2C x_2 + B\right),$$

$$\overline{B}_{22} = L, \overline{B}_{23} = M - x_2 L, \quad \overline{B}_{33} = L x_2^2 - 2M x_2 + N,$$

where

$$L = \frac{\alpha''}{2} - \frac{\alpha'\delta'}{4\delta} + \frac{\alpha}{2\delta}\left(\alpha'\gamma' - \beta^2\right) + \frac{\alpha^2}{\delta},$$

$$M = \frac{\beta''}{2} - \frac{\beta'}{4}\frac{\delta'}{\delta} + \frac{\beta}{2\delta}\left(\alpha'\gamma' - \beta'^2\right) + \frac{\alpha\beta}{\delta},$$

$$N = \frac{\gamma''}{2} - \frac{\gamma'}{4}\frac{\delta'}{\delta} + \frac{\gamma}{2\delta}\left(\alpha'\gamma' - \beta'^2\right) + \frac{\alpha\gamma}{\delta}.$$

The condition (4) is

$$2\delta'' - \frac{\delta'^2}{\delta} - \left(\alpha'\gamma' - \beta'^2\right) + 4\alpha = 0. \tag{10}$$

We consider the system of equations (3) as in § 3, taking up separately the five cases where $V$ is independent of $x_2$ and $x_3$; of $x_2$; of $x_1$; of $x_3$; involves all three variables. Only the first two cases lead to solutions.

For the first case we have the linear element of $S$

$$ds^2 = \frac{d\alpha^2}{4(a\alpha^{1/2} - \alpha)} + \alpha dx_2^2 - 2\alpha x_2 dx_2 dx_3 + \alpha(x_2^2 + 1) dx_3^2, \tag{11}$$

and

$$V = \sqrt{a\,\alpha^{-1/2} - 1},$$

where $a$ is a constant. This is the *longitudinal* solution obtained by Levi-Civita.[5]

For $V$ independent of $x_2$ two cases arise, according as $\beta$ can be made equal to zero by a transformation of coördinates, or not.

When $\beta = o$, the functions $\alpha$ and $\gamma$ must satisfy the three equations

$$\begin{aligned} &\alpha'' + \left(1 + \frac{1}{2k}\right)\frac{\alpha'\gamma'}{\gamma} - \left(1 + \frac{1}{k}\right)\frac{\alpha'^2}{2\alpha} + \frac{2\alpha}{\gamma}\left(1 + k\right) = 0, \\ &\gamma'' + \left(1 - \frac{1}{k}\right)\frac{\alpha'\gamma'}{2\alpha} + \frac{1}{2k}\frac{\gamma'^2}{\gamma} + 2\left(1 + k^2\right) = 0, \\ &2\,\alpha'\gamma' - \frac{\alpha'^2\gamma}{\alpha}\frac{1}{k} + \left(1 + \frac{1}{k}\right)\frac{\gamma'^2\alpha}{\gamma} + 4\,\alpha\left(k^2 + k + 1\right) = 0, \end{aligned} \tag{12}$$

where $k$ is any constant. For $k = 1$,

$$\gamma\,\gamma'^2 + 4\,\gamma^2 = 4\,a, \quad \frac{\alpha}{\gamma} = b\left(\frac{\sqrt{a} + \sqrt{a - \gamma^2}}{\gamma}\right)^{\pm\sqrt{3}}, \quad V = e^{x_1}\sqrt{\alpha\,\gamma} \tag{13}$$

where $a$ and $b$ are constants. Consequently,

$$ds^2 = \frac{4(a - \gamma^2)}{\gamma}\,d\gamma^2 + \alpha\,dx_2^2 - 2\,\alpha\,x_2\,dx_2\,dx_3 + \left(\alpha\,x_2^2 + \gamma\right)dx_3^2. \tag{14}$$

When $k \neq 1$, we solve the second of (12) for $\alpha'/\alpha$ and substitute in the third of (12). The first integral of the resulting equation is

$$\frac{\left(\gamma' + \sqrt{\gamma'^2 + 4\,k\,\gamma}\right)^b}{\left[(1 + k^2)\sqrt{\gamma'^2 + 4\,k\,\gamma} + b\,\gamma'\right]^{1+k^2}} = a\,\gamma^{k + \frac{b}{2}}, \tag{15}$$

where $a$ is an arbitrary constant and $b = \pm(1 - k)\sqrt{1 + k + k^2}$. Then we have

$$\begin{aligned} \alpha^{\frac{1}{k} - 1} &= \gamma^{\frac{1'}{k}}\left[(1 + k^2)\sqrt{\gamma'^2 + 4\,k\,\gamma} + b\,\gamma'\right]^2, \\ V &= e^{kx_3}\sqrt{\gamma}\left(\frac{\gamma}{\alpha}\right)^{\frac{1}{2k}}. \end{aligned} \tag{16}$$

When $\beta \neq 0$, we have

$$V = \delta^{1/4}\,e^{-2x_3}, \quad \delta = \alpha\,\gamma - \beta^2, \tag{17}$$

with the following conditions upon $\alpha$, $\beta$ and $\gamma$:

$$\begin{aligned} &\alpha'' - \frac{\alpha'\,\delta'}{4\delta} - \frac{\alpha}{2}\frac{\delta'^2}{\delta^2} - 14\frac{\alpha^2}{\delta} = 0, \\ &\beta'\alpha - \beta\alpha' = k\delta^{1/4}, \\ &\delta'^2 + 2\delta\,(\alpha'\,\gamma' - \beta'^2) + 24\,\alpha\,\delta = 0, \end{aligned} \tag{18}$$

where $k$ denotes an arbitrary constant.

Now (10) reduces to

$$\delta'' - \frac{1}{4}\frac{\delta'^2}{\delta} + 8\alpha = 0, \tag{19}$$

which is consistent with (18). Eliminating $\alpha$ from the first of (18) and (19), we get

$$\delta''' - \frac{3}{4}\frac{\delta'\delta''}{\delta} + \frac{5}{4}\frac{\delta''^2}{\delta} - \frac{21}{64}\frac{\delta'^4}{\delta^3} = 0.$$

If we introduce dependent and independent variables $t$ and $\theta$ by

$$\delta = e^{\int e^{\theta} dx_1}, \quad t = e^{-\theta}\frac{d\theta}{dx_1},$$

we get the equation

$$t\frac{d}{d\theta}\left(t\frac{dt}{d\theta}\right) + 7t^2\frac{dt}{d\theta} + 13\,t\frac{dt}{d\theta} + (2t+5)(3t+5)(t+3) = 0.$$

To the evident solutions of this equation, $t = -5/2$, $t = -5/3$, correspond the solutions:

$$\delta = 25x_1^{8/5}, \quad \alpha = -x_1^{-2/5}, \beta = bx_1^{9/5}, \gamma = -b^2x_1^4 - 25x_1^2,$$

$$\delta = 25x_1^{12/5}, \quad \alpha = -6x_1^{2/5}, \quad \beta = bx_1^{6/5}, \gamma = -\frac{1}{6}(b^2+25)x_1^2,$$

where $b$ is an arbitrary constant. For $t = -3$, we find $\alpha = 0$, which is excluded since we assume that $\delta > 0$ in accordance with the theory.

If we put $t\,dt/d\theta = y$, and take $y$ for dependent variable, the above equation may be replaced by

$$y\frac{dy}{dt} + y\left(7t+13\right) + \left(2t+5\right)\left(3t+5\right)\left(t+3\right) = 0.$$

When a solution of this equation is known, the corresponding functions $\alpha$, $\beta$, $\gamma$ can be found by quadratures.

5. Making use of the formulas of Bianchi[6] we find the following expressions for the principal curvatures of the spaces (7), (8), (11), (14)

$$-\underline{\overline{K}}_1 = (1+k)\underline{\overline{K}}_2 = \left(1+\frac{1}{k}\right)\underline{\overline{K}}_3 = \frac{k(k+1)^2}{(1+k+k^2)^2}\frac{1}{x_1^2}; \tag{7*}$$

$$\underline{\overline{K}}_1 = -\frac{\underline{\overline{K}}_2}{2} = \underline{\overline{K}}_3 = \frac{a}{2\alpha^{3/2}}; \tag{8*}$$

$$-\frac{\underline{\overline{K}}_1}{2} = \underline{\overline{K}}_2 = \underline{\overline{K}}_3 = \frac{a}{2\alpha^{3/2}}; \tag{11*}$$

$$\underline{\overline{K}}_1 = -\frac{a}{\gamma^3}(1+\sqrt{3}), \underline{\overline{K}}_2 = \frac{2a}{\gamma^3}, \underline{\overline{K}}_3 = \frac{-a}{\gamma^3}(1-\sqrt{3}). \tag{14*}$$

The principal curvatures for $k \pm 1$ in (12) can be obtained explicitly, but their forms are quite involved.

At each point of space the principal curvatures correspond to three directions, mutually perpendicular to one another. When the curves tangent to these directions are the curves of intersection of a triply-orthogonal system of surfaces, the space is called *normal* by Bianchi. All the spaces referred to above are normal. For the cases (7) and (8) the tangents to the curves of intersection $x_i = \text{const.}$, $x_j = \text{const.}$ are the principal directions.

For the cases of § 4 we put $x_2 = e^{\bar{x}_2 + z_2}$. Then for (11), the curves of intersection of surfaces $x_1 = \text{const.}$, $\bar{x}_2 = \text{const.}$, $x_3 = \text{const.}$ have the principal directions. For (14) and the case $k \neq 1$, the principal directions are given by $x_1 = \text{const.}$, $x_3 = \text{const.}$, and the orthogonal systems of curves on $\bar{x}_2 = \text{const.}$ defined by

$$\left(\alpha' - \frac{\alpha\gamma'}{\gamma}\right)dx_1^2 + 2\left[\alpha + \frac{1}{4}\alpha'\gamma' - \gamma\left(\frac{\alpha''}{2} - \frac{\alpha'^2}{4\alpha}\right)\right]dx_1\,dx_3 - \gamma\left(\alpha' - \frac{\alpha\gamma'}{\gamma}\right)dx_3^2 = 0.$$

[1] *Mem. Soc. Ital.*, 1896, p. 347.
[2] Levi-Civita, *Rend Lincei* (ser. 5), **26**, 1917, sem. 1 (460).
[3] Bianchi, *Lezioni*, **1**, 377; Cotton, *Ann. Fac. Toul.* (ser. 2), **1**, 1899 (410).
[4] *Science*, **54**, 1921 (305).
[5] *Rend. Lincei* (ser. 5), **27**, 1918, sem. 2 (350).
[6] *Lezioni*, **1**, 354.

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## *GEOMETRIC ASPECTS OF THE ABELIAN MODULAR FUNCTIONS OF GENUS FOUR (II)*

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8. *The form* $\binom{3}{1}\binom{2}{1}\binom{2}{1}$.—This form, written symbolically as $(\rho z)\,(rx)(sy)$, where $z$ is a point in $S_3$, $x$ a point in $S_2$, and $y$ a point in $S_2'$, has 36 coefficients and therefore $35-15-8-8=4$ absolute projective constants. Points $x$, $y$ determine a plane which becomes indeterminate for six pairs $x$, $y = p_i\,q_i$; $(i = 1, \ldots, 6)$ which form associated six points. They are the double singular points of a Cremona transformation $T$ of the fifth order between the planes $S_x$, $S_y$. A given plane $u$ is determined by $\infty^1$ pairs $x$, $y$ which lie, respectively, on the cubic curves, $(\rho\rho'\rho''u)\,(rx)\,(r'x)\,(r''x)\,(ss's'') = 0$, $(\rho\rho'\rho''u)\,(rr'r'')\,(sy)\,(s'y)\,(s''y) = 0$. These curves pass, respectively, through the six points $p_i$ and the six points $q_i$. Thus the given form is associated with a general cubic surface, $(\rho z)\,(\rho' z)\,(\rho'' z)\,(rr'r'')\,(ss's'') = 0$, with an isolated double-six of lines and separated

line-sixes. The mapping of the surface from the planes $S_x$ and $S_y$ is given by the above systems of cubics.

9. *The form* $\left(\begin{smallmatrix}1&1&2&2\\1&1&1&1\end{smallmatrix}\right)$.—In the space figure just described we insert a quadric $Q$ with generators $t$, $\tau$. The point coördinates $z$ can be replaced by bilinear expressions in $t$, $\tau$ and the form $\left(\begin{smallmatrix}3&2&2\\1&1&1\end{smallmatrix}\right)$ becomes a form $(p\tau)\ (\pi t)\ (\tau x)\ (sy)$ general of its type, with thirteen absolute constants corresponding to the four absolute constants of the above cubic surface and the nine additional constants for the inserted quadric. The quadric meets the cubic surface in a normal curve of genus 4 so that we have in space the figure of the normal sextic and a particular one of the $\infty^4$ cubic surfaces through it. In the planes $S_x$, $S_y$ we have projectively general (with thirteen absolute constants) sextics of genus 4 with nodes at $p_i$ and $q_i$, respectively, transforms of each other under $T$. The canonical adjoints of these sextics, with isolated $g_1^3$'s, are obtained by substituting for $u$ in 8 the proper bilinear expressions in $t$, $\tau$; while a similar substitution for $z$ in the equation of the cubic surface gives the equation on the quadric of the normal curve. We may therefore regard the general form $\left(\begin{smallmatrix}1&1&2&2\\1&1&1&1\end{smallmatrix}\right)$ as a definition of the projectively general plane sextic of genus 4.

10. *The forms* $F, \bar{F}$ *on the conic* $K(\tau)$. *Counter sextics.*—We mark the conic $K(\tau)$ on the plane $S_x$ and plot with reference to it the two counter sextics $\bar{S}_2(t)$ and $S_2(t)$. For each there is a cusp locus of perspective cubics $\bar{G}C(\tau)$ and $GC(\tau)$. Similarly we mark on a plane $S_y$ the conic $K(t)$ and plot with reference to it the two counter sectics $\bar{S}_1(\tau)$, $S_1(\tau)$ which are paired with the above counter sextics, and which have for cusp loci of perspective cubics the sextic curves $GC(t)$, $\bar{G}C(t)$, respectively.

We now consider the form $F = (\alpha t)^3\ (a\tau)^3 = 0$ where $\tau$ is a tangent of $K(\tau)$. For variable $t$ we have $\infty^1$ triangles circumscribed about $K(\tau)$ whose vertices run over a sextic curve. If $t$ determines $\tau_1$, $\tau_2$, $\tau_3$ then the point $t$, $\tau_1$, $\tau_2$, of this sextic curve is birationally related to the solution $t$, $\tau_3$ of $F = 0$. Hence the sextic curve is birationally equivalent to $GC(\tau)$ and as a result of the algebraic discussion of the next two sections we prove that this sextic curve is actually $GC(\tau)$. This amounts to the effective elimination of $t$ from $(a\tau_1)^3\ (\alpha t)^3 = 0$, $(a\tau_2)^3\ (\alpha t)^3 = 0$ and the separation of the factor $(\tau_1\tau_2)^3$. Hence we have on the sextic $GC(\tau)$ two $g_1^3$'s such that the $\infty^1$ $t$-triads are cusp triangles of perspective cubics of $S_2(t)$ and the $\infty^1$ $\tau$-triads are triangles circumscribed to $K(\tau)$ for which the intersection of the lines joining each vertex to the contact of the opposite side is the point $t$ of $\bar{S}_2(t)$. Of course a similar statement is true of any one of the four cusp loci.

If we polarize the form $F$ into $(a\tau)\ (a\tau_1)\ (a\tau_2)\ (\alpha t)\ (\alpha t_1)\ (\alpha t_2)$ and replace the pairs $\tau_1, \tau_2$; $t_1, t_2$ by points $x; y$ referred in Darboux coördinates to the conics $K(\tau)$; $K(t)$ in $Sx$; $Sy$, respectively, we have a form $(\pi x)\ (d\tau)\ (\delta t)\ (\rho y)$ of the type discussed in 9. Hence the sextics $GC(\tau)$ and $GC(t)$ are trans-

forms of each other under the quintic Cremona transformation $T$ but on the two the role of the $t$ and $\tau$ triads with reference to the rational sextics and conics $K$ is reversed. It may be shown that the projective peculiarity (equivalent to four conditions) of our birationally general sextics $\bar{G}C(\tau)$ and $GC(\tau)$ is that their $\tau$ triads envelop a conic $K(\tau)$. This same property belongs to the $t$-triads after transformation of $GC(\tau)$ by $T$ (or $\bar{G}C(\tau)$ by the corresponding $\bar{T}$) into a sextic in the plane $S_y$.

We have mentioned in I 7 certain intersectional properties of the conic $\bar{b}$, the rational sextic $\bar{S}_2(t)$, and the cusp locus $\bar{G}C(\tau)$. Analogous results for the conic $K(\tau)$, the sextic $\bar{S}_2(t)$, and the locus $GC(\tau)$ are as follows. The conic $K(\tau)$ meets $\bar{S}_2(t)$ in 12 points whose parameters $t$ on $\bar{S}_2(t)$ are the branch points of the function $\tau(t)$ determined by $F = 0$, and whose parameters $\tau$ on $K(\tau)$ are the branch point values of this function. The 12 residual function values $\tau$ furnish 12 common tangents of $K(\tau)$ and $GC(\tau)$, i.e., the tangents to $GC(\tau)$ at the 12 coincidences of $g_1^3(\tau)$ upon it touch $K(\tau)$. The remaining common tangents of $K(\tau)$ and $GC(\tau)$ are double tangents of $GC(\tau)$ whose parameters on $K(\tau)$ are the branch points of the function $t(\tau)$ in $F = 0$. The two further tangents to $K(\tau)$ from the contacts with $GC(\tau)$ of each double tangent meet in a point which is a coincidence of $g_1^3(t)$ on $GC(\tau)$.

11. *The reciprocity between the forms $F$, $\bar{F}$; algebraic discussion.*—We denote by $A$ the determinant of the coefficients (without binomial factors) of the form $F$; the coefficients of $F$ are then, to within numerical factors, three-row minors of $A$. The reciprocity between the forms $F$, $\bar{F}$ is then brought out by the fact that the covariant $\bar{F}$ formed for $\bar{F}$ as a ground form is again $F.A^2/_3{}^4$. The invariant $(aA)^3\,(\alpha A)^3$ is $4\,A$. This reciprocity fails when $A$ vanishes. Then the four cubics in $t$ furnished by $F$ are linearly dependent. When expressed in terms of three the coefficients in $F$ are three cubics in $\tau$, and the form $F = 0$ can be interpreted as the incidence condition of point $\tau$ on a rational *plane* cubic curve $C_1$ with line $t$ of a rational *plane* cubic envelope $C_2$. In this case the form $\bar{F}$ factors into two cubics $(A\tau)^3.(At)^3$, the conjugate cubics of the rational curves $C_1$, $C_2$. Now all the (usually non-vanishing) odd transvectants of the double form $\bar{F}$ will vanish whence in the general case they contain the factor $A$ and are of the second degree for $F$. From their degree and orders they can be identified at once with ${}^2/_9\,(b\tau)^4\,(\beta t)^4 \equiv (aa')\,(\alpha\alpha')\,(a\tau)^2\,(a'\tau)^2\,(\alpha t)^2\,(\alpha' t)^2$; $8\,\delta \equiv (aa')^2\,(\alpha\alpha')^2$; ${}^4/_3\,(\gamma t)^4 \equiv (aa')^3\,(\alpha\alpha')\,(\alpha t)^2\,(\alpha' t)^2$; ${}^4/_3(c\tau)^4 = (\alpha\alpha')^3\,(aa')\,(a\tau)^2\,(a'\tau)^2$. With respect to the original space cubics $C_1, C_2$ these forms are interpreted as follows. The form $(b\tau)^4\,(\beta t)^4$ vanishes when tangent $\tau$ of $C_1$ meets tangent $t$ of $C_2$; $\delta = 0$ if the null systems of $C_1$, $C_2$ are apolar; $(\gamma t)^4$ or $(c\tau)^4$ vanishes if the tangent $t$ of $C_2$ or the tangent $\tau$ of $C_1$ is a line of the null system of the other curve.

The reciprocity between the dual forms $F$, $\bar{F}$ is due algebraically to

the fact that their coefficients are, respectively, the one- and three- row minors of $A$. The four comitants of the second degree above are evidently self-dual in meaning and therefore should involve the two-row minors of $A$. Indeed we find that the 36 coefficients of the four comitants $(b\tau)^4$ $(\beta t)^4$, $(\gamma t)^4$, $(c\tau)^4$, $\delta$ are linearly independent in the 36 two-row minors of $A$. The only remaining comitants of the second degree, not linear in the minors of $A$, are the two sextics $S_1(\tau)$, $S_2(t)$.

The 36 two-row minors of a four-row determinant $A$, though linearly independent, must satisfy a system of quadratic relations. It is not hard to see that there are precisely 41 such quadratic relations. If we take the above four comitants and form all of *their* comitants of total second degree in the coefficients of the four we have a set of comitants with 666 coefficients which is the number of quadratic combinations of the 36 minors. Hence the coefficients of this new set of comitants must be connected by a system of 41 linear relations due to the existence of the 41 quadratic relations among the minors. These relations will be expressed by the existence of a system of syzygies of the second degree in the coefficients of the four comitants. We find that this system of syzygies is $(b\tau)^4$ $(b'\tau)^4$ $(\beta\beta')^4 = 6[(c\tau)^4]^2$; $(bb')^4$ $(\beta t)^4$ $(\beta' t)^4 = 6[(\gamma t)^4]^2$; $(bb')^2$ $(b\tau)^2$ $(b'\tau)^2$ $(\beta\beta')^4 = 6(cc')^2$ $(c\tau)^2$ $(c'\tau)^2$; $(bb')^4$ $(\beta\beta')^2$ $(\beta t)^2$ $(\beta' t)^2 = 6(\gamma\gamma')^2$ $(\gamma t)^2$ $(\gamma' t)^2$; $(b\tau)^4$ $(\beta\gamma)^4 = 6\,\delta.(c\tau)^4$; $(bc)^4$ $(\beta t)^4 = 6\,\delta.(\gamma t)^4$; $(bb')^4$ $(\beta\beta')^4 = 6(\gamma\gamma')^4 = 6(cc')^4 = 36\,\delta^2$.

The 41 coefficients of these syzygies furnish the quadratic relations. The syzygies themselves determine for given $(b\tau)^4$ $(\beta t)^4$ the three remaining forms $(c\tau)^4$, $(\gamma t)^4$, $\delta$ to within a change of sign of any two.

12. *The fundamental combinants of $F$, $\overline{F}$.* We have had occasion at times to consider such pencils of cubics in $t$ as are determined by the members $(a\tau_1)^3$, $(\alpha t)^3$, $(a\tau_2)^3$ $(\alpha t)^3$ $(\tau_1 \neq \tau_2)$. If $t_1$, $t_2$ belong to the same member of this pencil then the fundamental combinant of Gordan for the pencil is

$$\Gamma = \begin{vmatrix} (a\tau_1)^3 & (\alpha t_1)^3 & (a\tau_2)^3 & (\alpha t_1)^3 \\ (a'\tau_1)^3 & (\alpha' t_2)^3 & (a'\tau_2)^3 & (\alpha' t_2)^3 \end{vmatrix}.$$

Evidently it expresses also that $\tau_1$, $\tau_2$ belong to the same member of the pencil determined by the members $(a\tau)^3$ $(\alpha t_1)^3$ $(a\tau)^3$ $(\alpha t_2)^3$ $(t_1 \neq t_2)$. We call $\Gamma$ the fundamental combinant of $F$, and $\overline{\Gamma}$ the corresponding fundamental combinant of $\overline{F}$. Each is expressible in terms of the two-row minors of $A$ and therefore in terms of the four comitants of 11. We find that $\Gamma$, $\overline{\Gamma} = (\tau_1\tau_2).(t_1t_2)\left\{(b\tau_1)^2\,(b\tau_2)^2\,(\beta t_1)^2\,(\beta t_2)^2 \pm (t_1t_2)^2.(c\tau_1)^2\,(c\tau_2)^2 \pm (\tau_1\tau_2)^2.(\gamma t_1)^2\,(\gamma t_2)^2 + (t_1t_2)^2.(\tau_1\tau_2)^2.\delta\right\}$

where the upper sign holds for $\Gamma$ and the lower sign (together with the factor $A/9$) for $\overline{\Gamma}$. Let us call the four terms within this brace, taken with plus signs, $K$, $L$, $M$, $N$, respectively. Then the involution of binary cubics determined in 7 by the form $(\pi x)$ $(d\tau)$ $(\delta t)^3$ for variable $\tau$ has for fundamental combinant (when $x$ is replaced by $\tau_1$, $\tau_2$) $K + L - M - N$

and the corresponding combinant for the counter sextic $S_2(t)$ rather than $\bar{S}_2(t)$ is $K-L+M-N$. These four involutions all arise from any one by changing the signs of any two of the comitants $(\gamma t)^4$, $(c\tau)^4$, $\delta$. In addition to the facts derived in 10 from this set of involutions other results are easily read off from their simple form. For example the conic $\bar{b}$ defined in 7 for $\bar{S}_2(t)$ and the corresponding conic $b$ for $S_2(t)$ are in a pencil with the conic $K(\tau)$ and meet $K(\tau)$ in the four points determined by $(c\tau)^4 = 0$.

# LIST OF PUBLICATIONS OF THE NATIONAL ACADEMY OF SCIENCES AND OF THE NATIONAL RESEARCH COUNCIL

## PART 1. PUBLICATIONS OF THE NATIONAL ACADEMY OF SCIENCES

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Forty-fifth Congress, 3rd Session, House of Representatives Misc. Doc. no. 5. Surveys of the Territories. Letters from the Acting President of the National Academy of Sciences, transmitting a report on the surveys of the Territories. Ordered printed, December 3, 1878. 8°. Pp. 1–27.

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Forty-ninth Congress, 1st Session. Senate, Ex. Doc. no. 67. Letter from the Secretary of the Navy, transmitting, in compliance with the Senate resolution, February 2, 1886, report of the National Academy of Sciences upon the proposed new Naval Observatory. Ordered printed, February 10, 1886.

Report on the organization of the National Surveys and the Signal Service. In Senate Misc. Doc. no. 82, 49th Congress, 1st Session. Pp. 1–37. Ordered printed, March 16, 1886. 1886.

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| 1870 | | | 1890 | 36 | 1891 | 1909 | 43 | 1910 |
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| 1876 | | | 1896 | 39 | 1897 | 1915 | 74 | 1916 |
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